

Connection between neutrino mass models and muon experiments

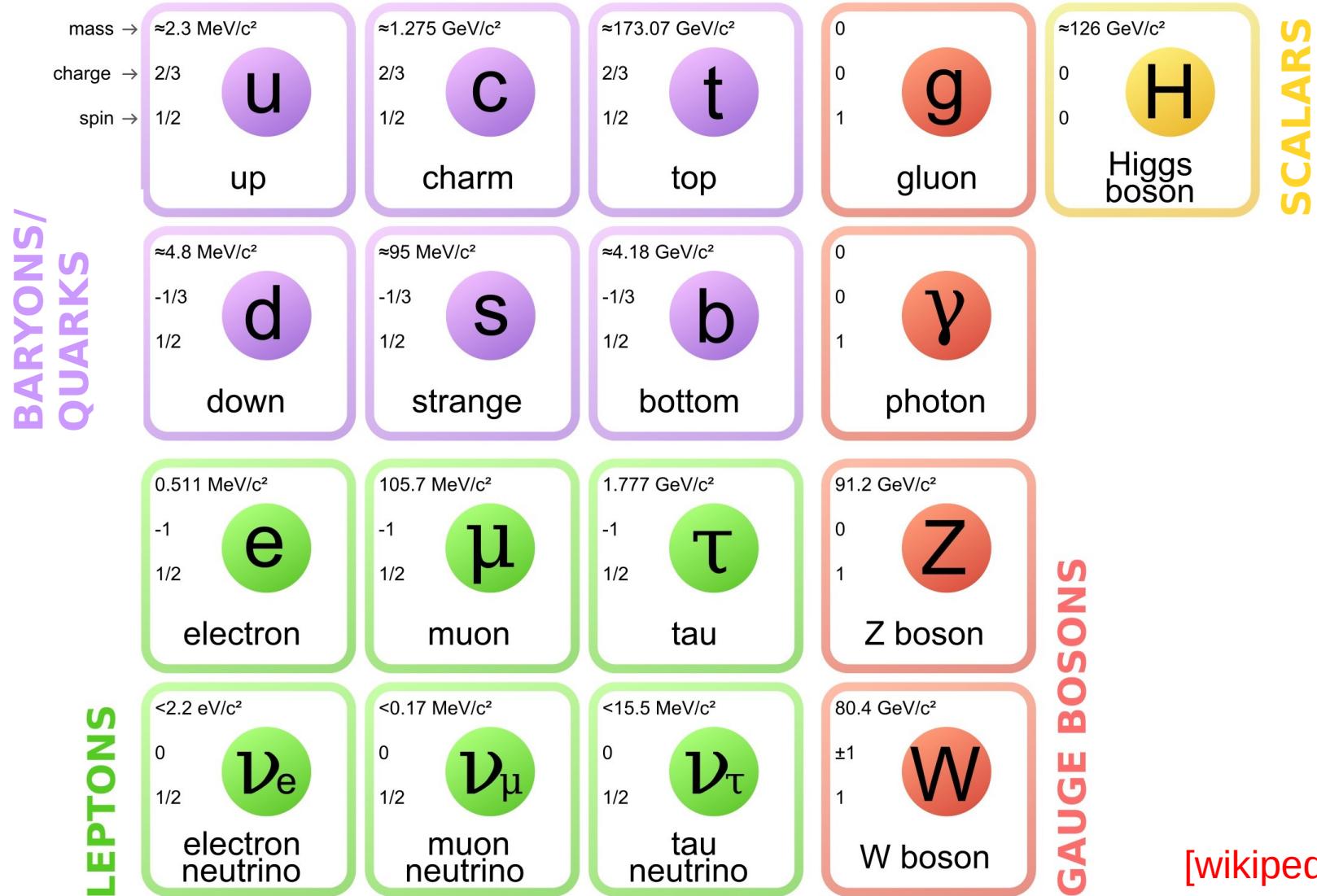
Julian Heeck

NuFact 2022, Utah, USA

08/05/2022

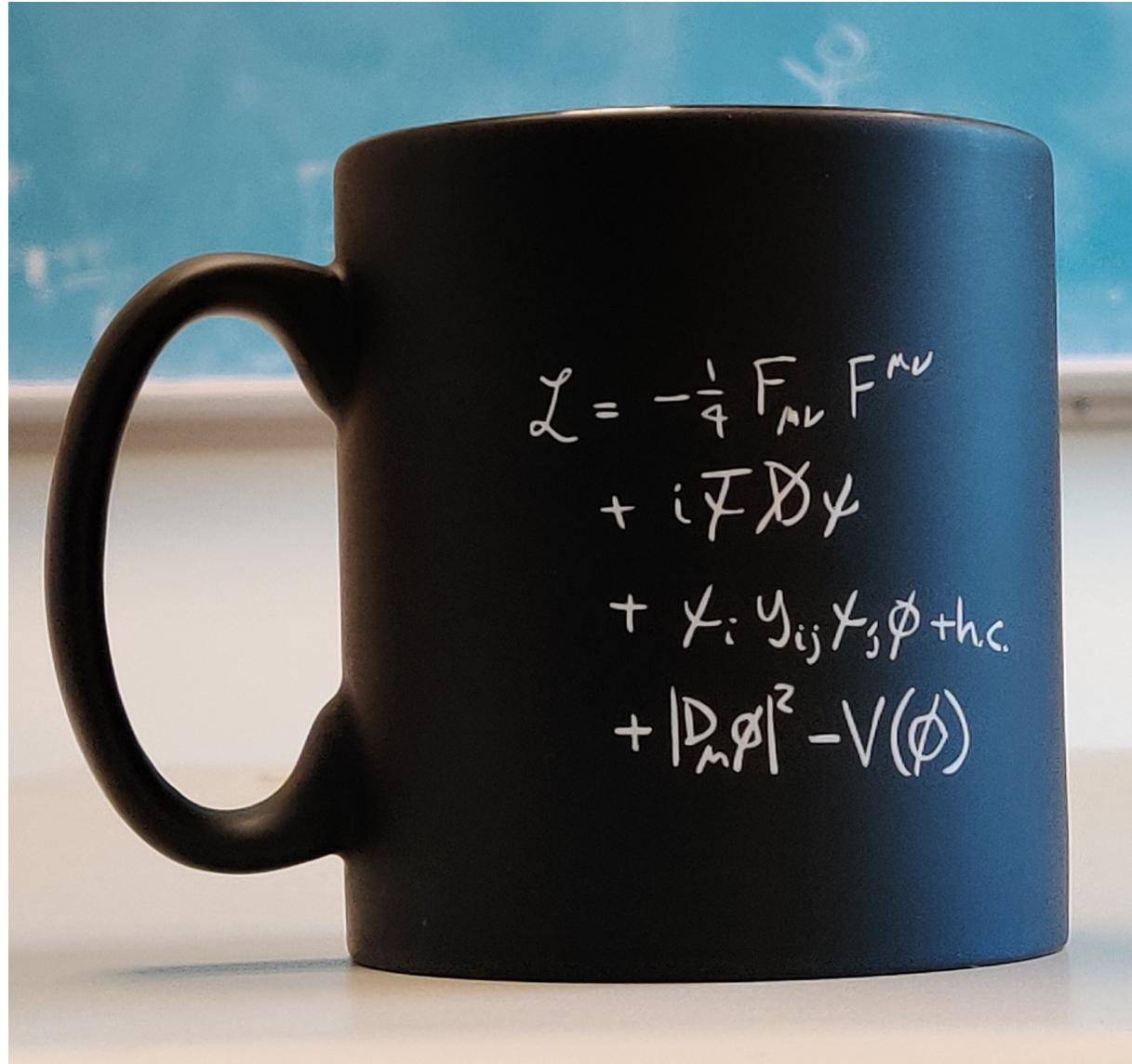


Elementary particles



[wikipedia]

The Standard Model



Symmetries of the Standard Model

- Rephasing lepton and quark fields:

$$\begin{aligned} U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \\ = \\ \cancel{U(1)_{B+L}} \times U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e} . \end{aligned}$$

↑

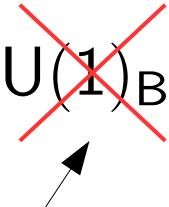
- Broken non-perturbatively, but unobservable. [*'t Hooft, PRL '76*]
- True accidental global symmetry:

$$U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e} .$$

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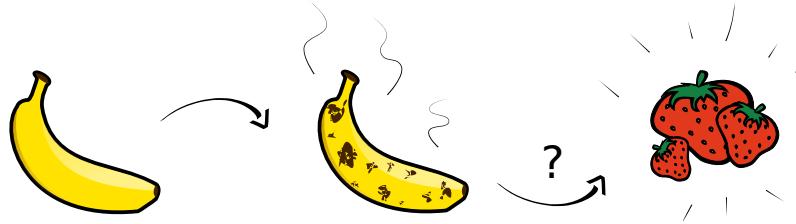
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$$U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e} .$$

Lepton flavor conservation!

Prediction of Standard Model.

Flavor violating decays

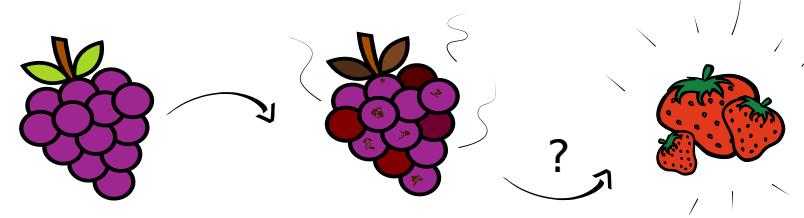


- Prime example: $\mu \rightarrow e\gamma$ @ MEG.
- Observation = new particles.
- $\mu \rightarrow e$ conversion @ Mu2e can probe scales up to 10^7 GeV.

See talks by Palo, Perrevoort,
Group, Yamamoto, Dekkers,
Papa, Lynch, Kriewald, Oksuzian.

LFV	process	current	future	exp
$ \Delta L_\mu = 1$	$\mu \rightarrow e\gamma$	4.2×10^{-13}	6×10^{-14}	MEG-II
$ \Delta L_\mu = \Delta L_e $	$\mu \rightarrow e\bar{e}e$	1.0×10^{-12}	10^{-16}	Mu3e
$ \Delta L_e = \Delta L_\mu $	$\mu \rightarrow e$ conv.	$\mathcal{O}(10^{-12})$	10^{-16}	Mu2e, COMET
$ \Delta L_e $	$h \rightarrow e\bar{\mu}$	6.1×10^{-5}	10^{-5}	LHC
$ \Delta L_e $	$Z \rightarrow e\bar{\mu}$	7.5×10^{-7}	10^{-10}	FCC-ee
$ \Delta L_e $	had $\rightarrow e\bar{\mu}$ (had)	4.7×10^{-12}	10^{-12}	NA62

Flavor violating decays

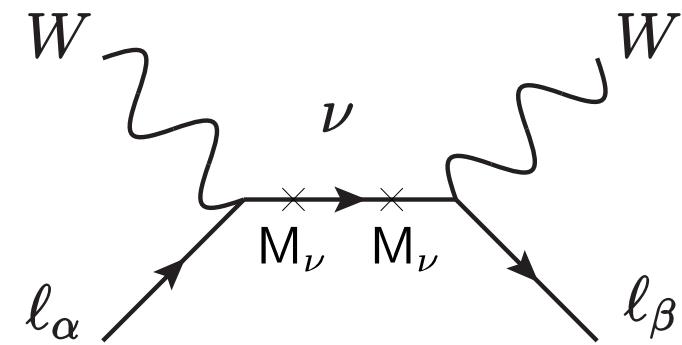


- Produce tauons at B factories (BaBar, Belle (II), LHCb).
- Observation = **new particles**.
- $\tau \rightarrow e^- e^+ e^-$ @ Belle II will probe scales up to $2 \times 10^4 \text{ GeV}$.

LFV	process	current	future	exp
$\frac{1}{\Delta L_\tau}$	$\tau \rightarrow e\gamma$	3.3×10^{-8}	10^{-9}	Belle II
$\frac{1}{\Delta L_e}$	$\tau \rightarrow e\bar{\ell}\ell$	2.7×10^{-8}	10^{-9}	Belle II
$\frac{1}{\Delta L_e}$	$\tau \rightarrow e \text{ had}$	$\mathcal{O}(10^{-8})$	10^{-9}	Belle II
$\frac{1}{\Delta L_e}$	$h \rightarrow e\bar{\tau}$	4.7×10^{-3}	10^{-4}	LHC
$\frac{1}{\Delta L_e}$	$Z \rightarrow e\bar{\tau}$	9.8×10^{-6}	10^{-9}	FCC-ee
$\frac{1}{\Delta L_e}$	$\text{had} \rightarrow e\bar{\tau}(\text{had})$	$\mathcal{O}(10^{-6})$	–	Belle II

Neutrino oscillations = flavor violation

- Observations of $\nu_\alpha \rightarrow \nu_\beta$ prove that $M_\nu \neq 0$ and $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$ is **broken!**
- Amplitudes for **charged lepton flavor violation** are suppressed:



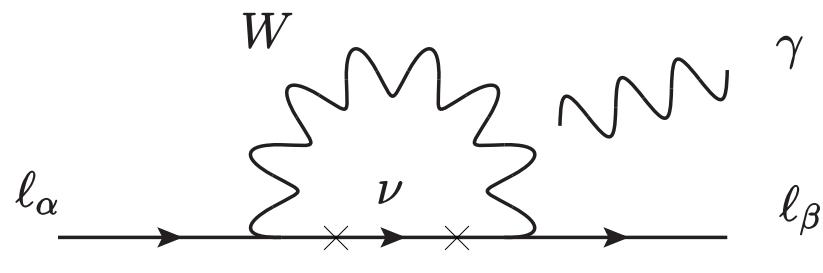
$$\mathcal{A}(\ell_\alpha^- \rightarrow \ell_\beta^-) \propto \frac{(M_\nu M_\nu^\dagger)_{\alpha\beta}}{M_W^2} < 10^{-24}.$$

Great goalpost for
Snowmass 3000!

- Most (neutrino mass) models also generate CLFV rates **unsuppressed** by M_ν that could be observable.

Neutrino mass \Rightarrow charged LFV?

- SM + Dirac neutrinos: $L = L_{\text{SM}} - (y \bar{\nu} H \nu_R + \text{h.c.}) + i \bar{\nu}_R \partial^\mu \nu_R$



$$\begin{aligned}
 m_\nu &= y \langle H \rangle \\
 &= U \text{diag}(m_1, m_2, m_3) V_R \\
 &\stackrel{!}{\lesssim} eV
 \end{aligned}$$

- All CLFV is GIM suppressed:

$$\frac{\Gamma(\ell_\alpha \rightarrow \ell_\beta \gamma)}{\Gamma(\ell_\alpha \rightarrow \ell_\beta \nu_\alpha \bar{\nu}_\beta)} \simeq \frac{3\alpha_{\text{EM}}}{32\pi} \left| \sum_{j=2,3} U_{\alpha j} \frac{\Delta m_{j1}^2}{M_W^2} U_{j\beta}^\dagger \right|^2 < 5 \times 10^{-53}.$$

[‘77: Petcov; Bilenky, Petcov, Pontecorvo; Marciano, Sanda; Lee, Pakvasa, Shrock, Sugawara]

Seesaw mass \Rightarrow charged LFV?

- SM + seesaw neutrinos: $L = L_{\text{SM}} + i \bar{N}_R \not{D} N_R - (\frac{1}{2} M_R \bar{N}_R^c N_R + y \bar{L} H N_R + \text{h.c.})$
- Violates $\Delta L = 2$. For large M_R : $m_D \bar{\nu}_L N_R$

$$M_N \simeq M_R, \quad M_\nu \simeq -m_D M_R^{-1} m_D^\top = U^* \text{diag}(m_1, m_2, m_3) U^\dagger.$$

- Majorana neutrinos!
 - LFV:
$$\frac{\Gamma(\ell_\alpha \rightarrow \ell_\beta \gamma)}{\Gamma(\ell_\alpha \rightarrow \ell_\beta \nu_\alpha \bar{\nu}_\beta)} \simeq \frac{3\alpha_{\text{EM}}}{8\pi} |(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta}|^2.$$
- $\mathcal{O}(M_\nu^4/m_D^4)$
- Not true with fine-tuning or structure in m_D .

Seesaw parameters

$$L = L_{SM} + i\bar{N}_R \not{\partial} N_R - (\frac{1}{2} M_R \bar{N}_R^c N_R + m_D \bar{\nu}_L N_R + h.c.)$$

$$\Rightarrow M_\nu \simeq -m_D M_R^{-1} m_D^T \quad \& \quad BR(\ell_\alpha \rightarrow \ell_\beta \gamma) \propto |(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta}|^2.$$

- One to one correspondence

$$\{m_D, M_R\} \leftrightarrow \{M_\nu, m_D M_R^{-2} m_D^\dagger\}.$$

[Broncano, Gavela, Jenkins,
hep-ph/0210271]

- Or: unique d=6 operator $(y M_R^{-2} y^\dagger)(\bar{L} H)(i\not{\partial})(H^\dagger L)$.
- Gives LFV and non-unitary PMNS.

LFV complementary to M_ν !

EFT Seesaw

- One to one correspondence

$$\{m_D, M_R\} \leftrightarrow \{M_\nu \simeq -m_D M_R^{-1} m_D^T, m_D M_R^{-2} m_D^\dagger\}.$$

- Flavor structure of m_D to get large LFV and tiny M_ν :

$$m_D \propto \begin{pmatrix} 1 \\ z\sqrt{\frac{M_2}{M_1}} \\ \pm i\sqrt{1+z^2}\sqrt{\frac{M_3}{M_1}} \end{pmatrix} (\lambda_e \ \lambda_\mu \ \lambda_\tau) + \epsilon$$

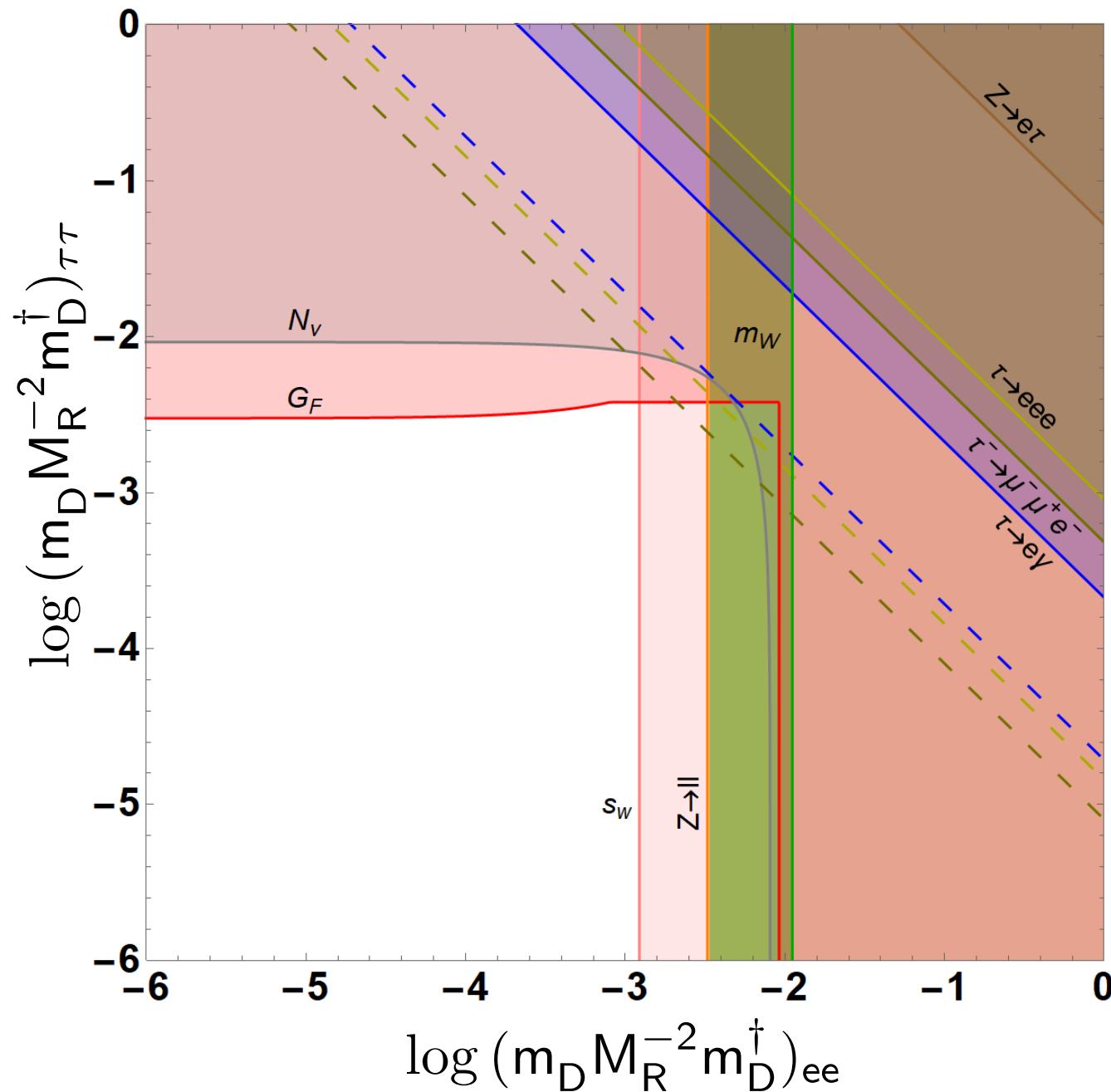
[Coy & Frigerio, 1812.03165;
See also Kersten & Smirnov '07]

- LFV matrix only has 3 real free parameters:

$$(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta} = \sqrt{(m_D M_R^{-2} m_D^\dagger)_{\alpha\alpha} (m_D M_R^{-2} m_D^\dagger)_{\beta\beta}}.$$

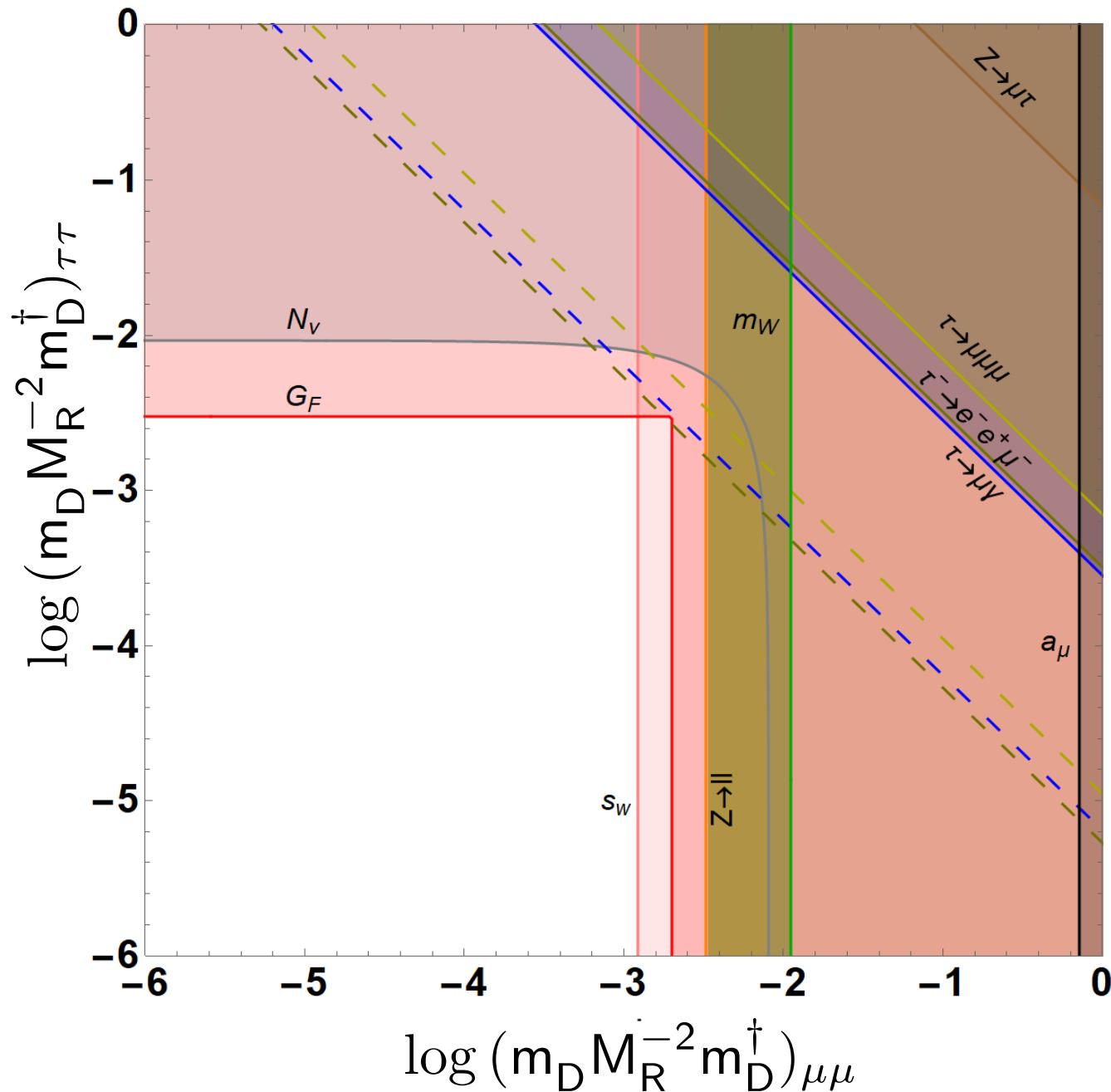
- Relation between LFV and non-LFV constraints.

$$(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta} = \sqrt{(m_D M_R^{-2} m_D^\dagger)_{\alpha\alpha} (m_D M_R^{-2} m_D^\dagger)_{\beta\beta}}.$$



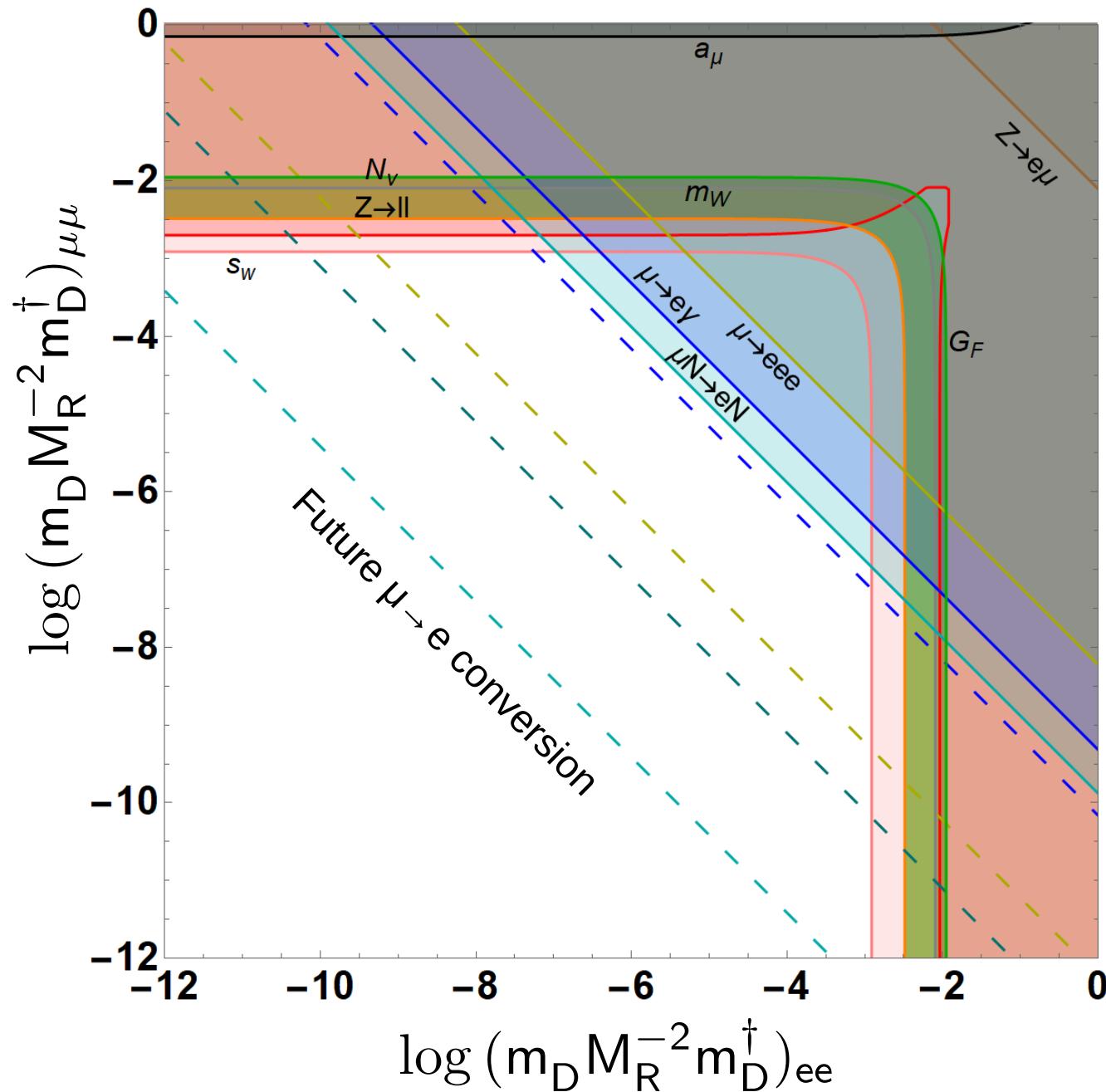
[Coy & Frigerio, 1812.03165 & 2110.09126]

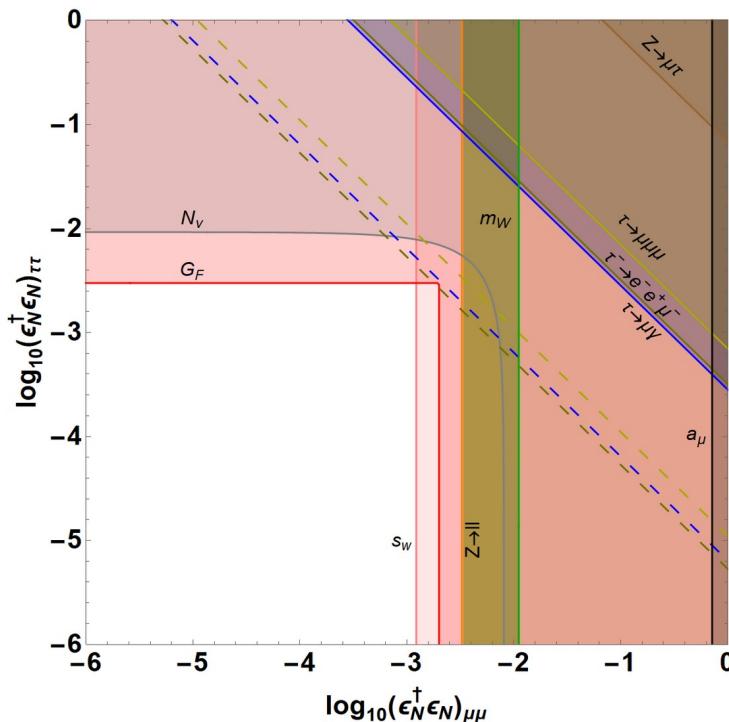
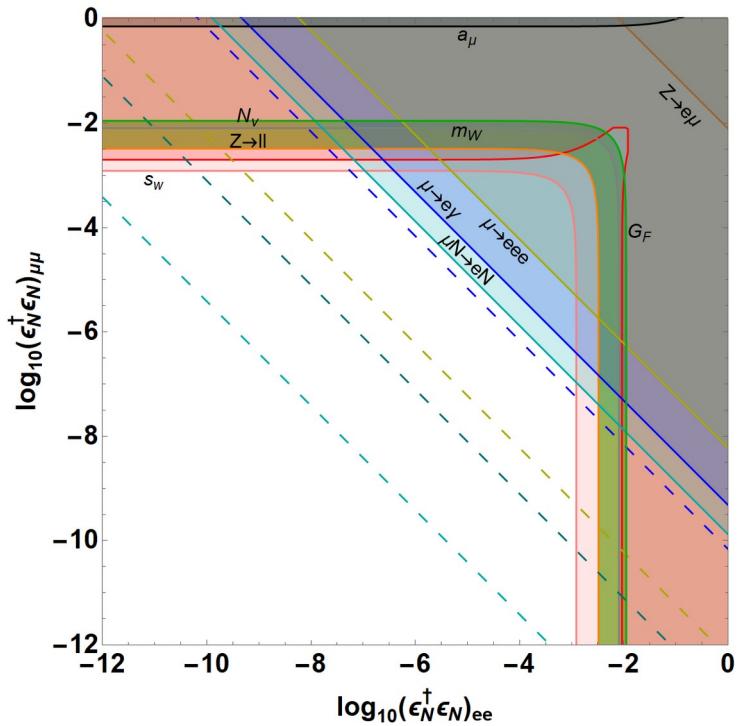
$$(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta} = \sqrt{(m_D M_R^{-2} m_D^\dagger)_{\alpha\alpha} (m_D M_R^{-2} m_D^\dagger)_{\beta\beta}}.$$



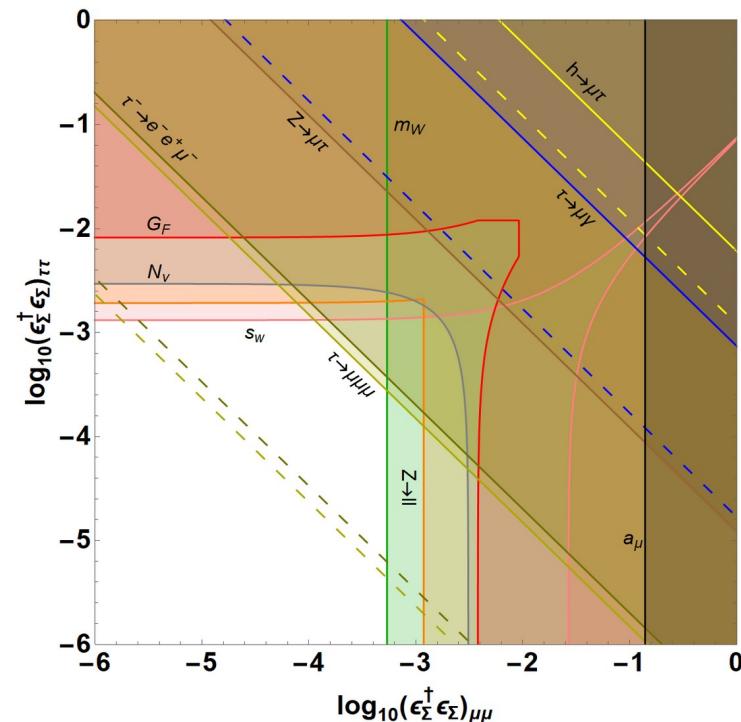
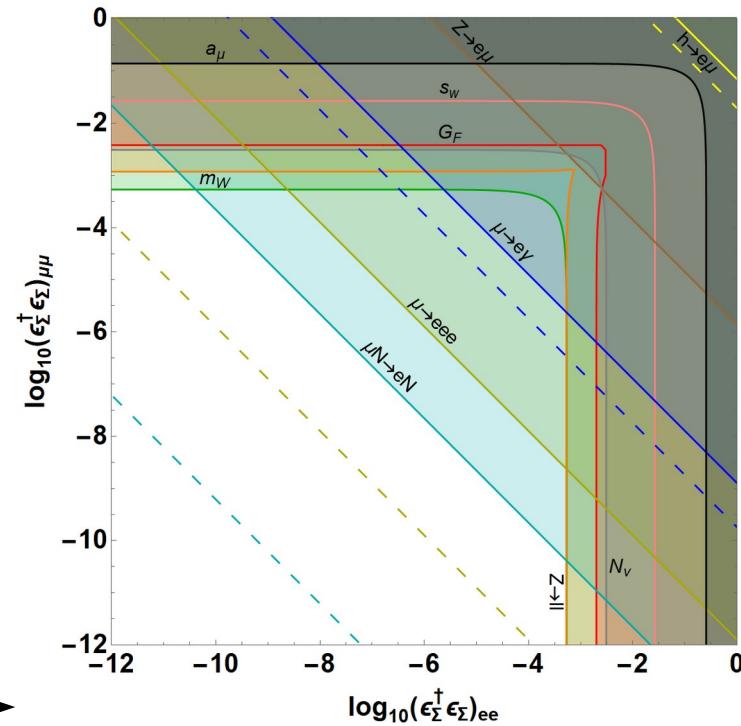
[Coy & Frigerio, 1812.03165 & 2110.09126]

$$(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta} = \sqrt{(m_D M_R^{-2} m_D^\dagger)_{\alpha\alpha} (m_D M_R^{-2} m_D^\dagger)_{\beta\beta}}.$$



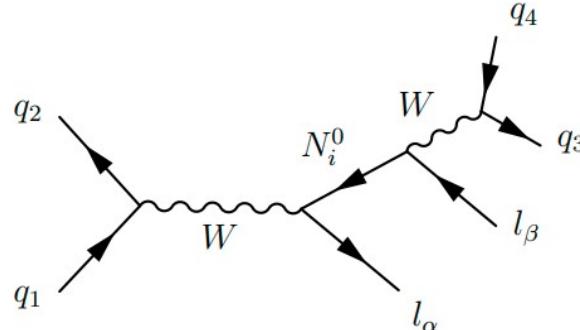


Type I →
Type III
seesaw



Seesaw

- Interesting interplay of LFV and non-LFV observables.
- Type-I can get large rates for muon LFV, tauons difficult.
[Crivellin, Kirk, Manzari, 2208.00020 find testable tau LFV though]
- Type-III seesaw can also give large tau LFV. [Coy & Frigerio '22]
- For “light” right-handed neutrinos EFT breaks down:
 - N_R^0 can be emitted on-shell at colliders.
 - Allows better access to seesaw parameters.
See talks by ATLAS & CMS later.



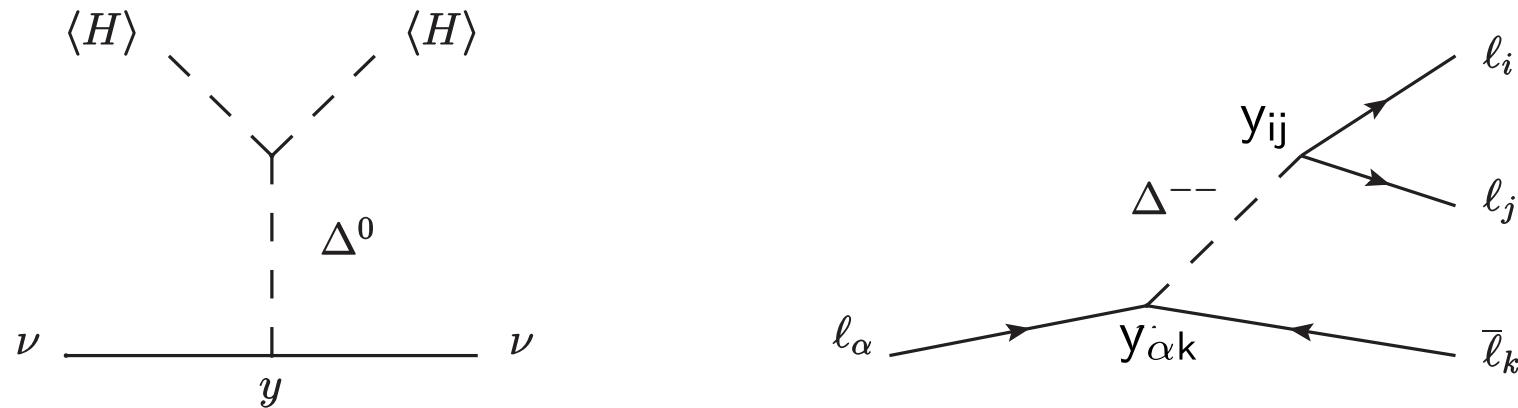
[Kersten & Smirnov '07]

LFV complementary to M_ν !

Scalar-triplet (type-II) seesaw

[Konetschny & Kummer '77; Magg & Wetterich, '80; Schechter & Valle '80; Cheng & Li, '80; Mohapatra & Senjanovic, '81]

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + |\mathbf{D}_\alpha \Delta|^2 - (y_{\alpha\beta} \bar{\mathbf{L}}_\alpha^c \Delta \mathbf{L}_\beta + \mu \mathbf{H} \Delta \mathbf{H} + \text{h.c.})$$

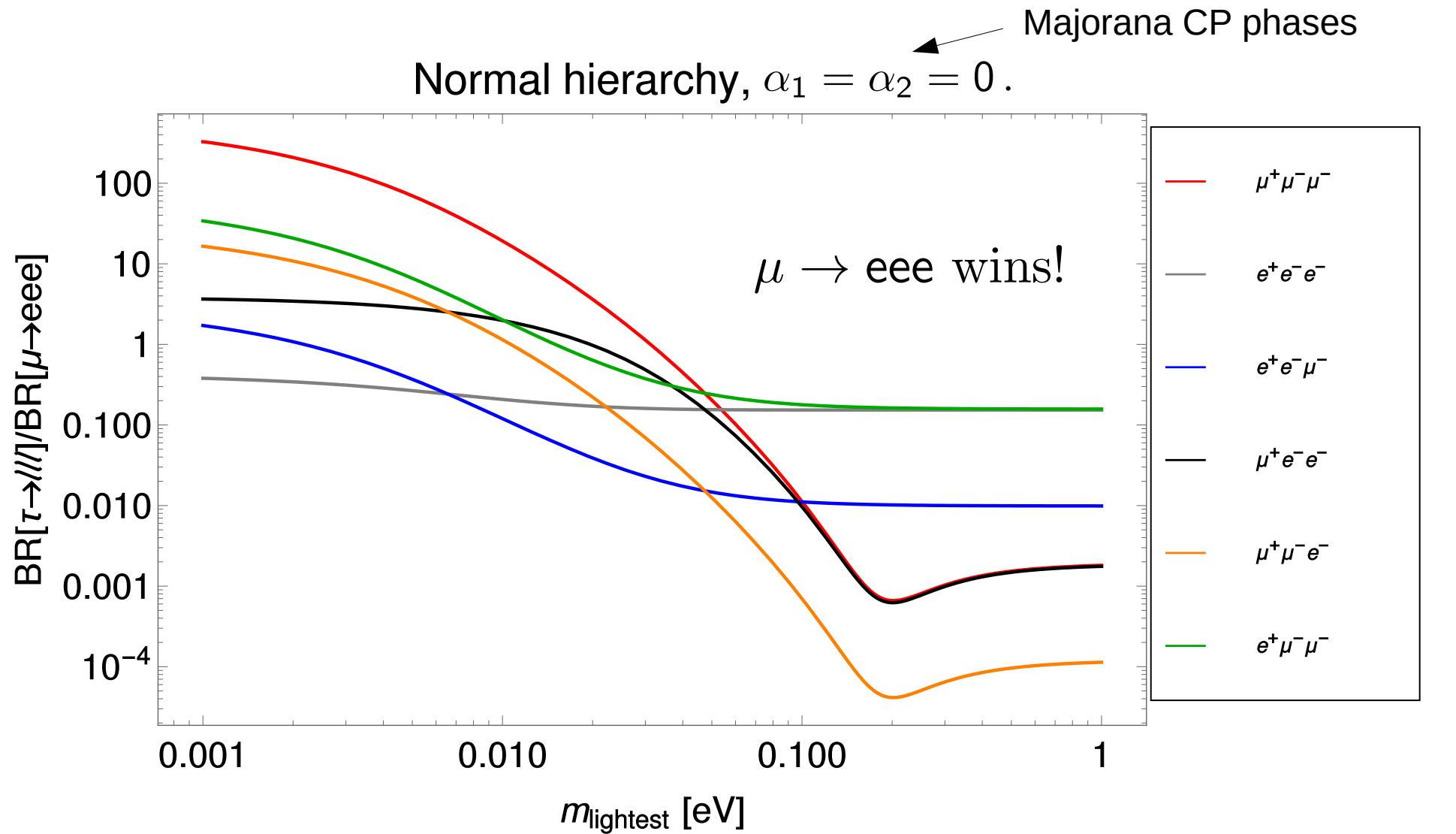


$$\Rightarrow (\mathbf{M}_\nu)_{\alpha\beta} \simeq y_{\alpha\beta} \frac{2\mu v^2}{M_\Delta^2} \quad \& \quad \text{BR}(\ell_\alpha \rightarrow \ell_i \ell_j \bar{\ell}_k) \propto |(\mathbf{M}_\nu)_{\alpha k}|^2 |(\mathbf{M}_\nu)_{ij}|^2.$$

[Pich, Santamaria, Bernabeu, '84; Abada++, 0707.4058]

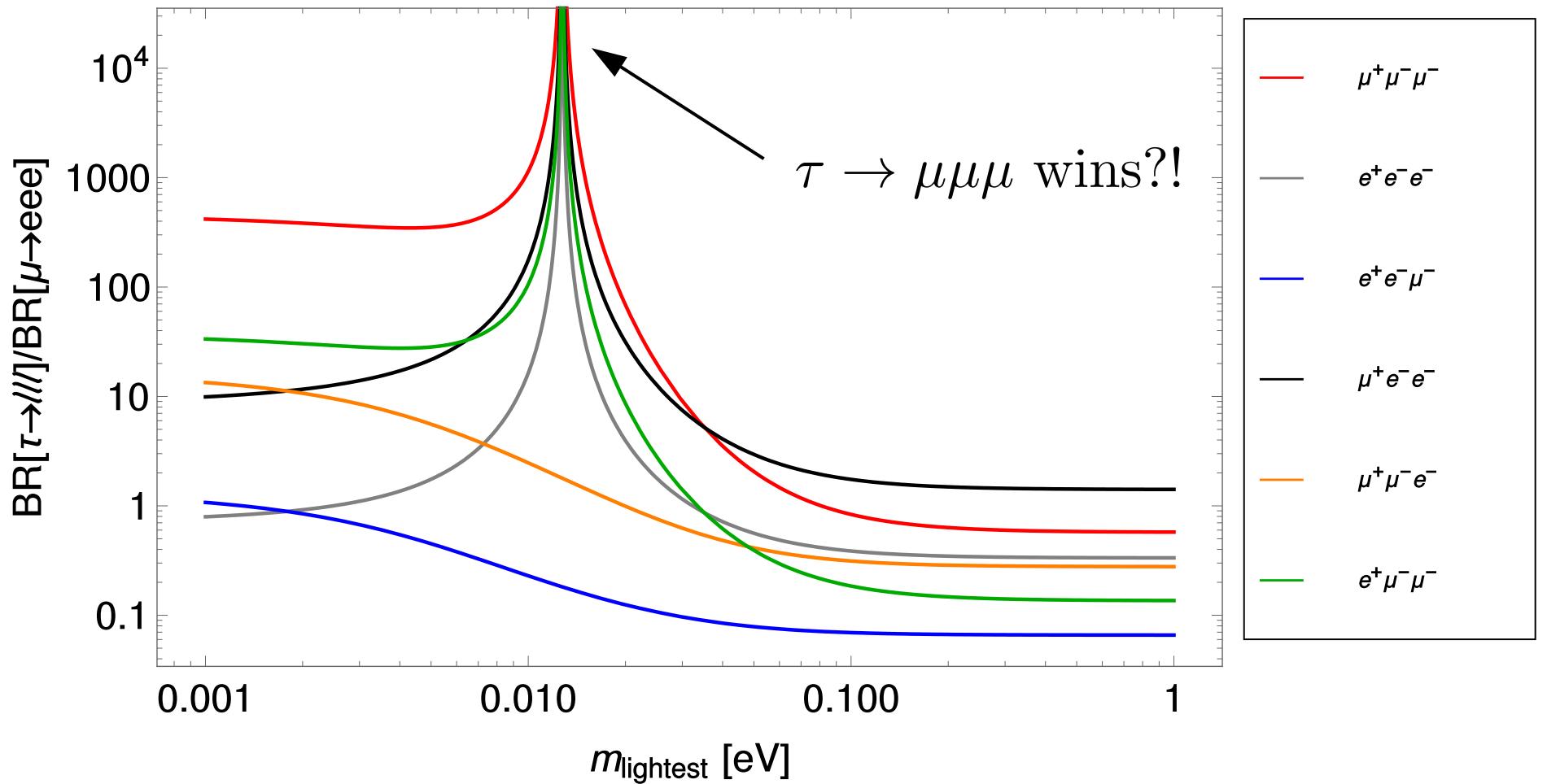
Prediction of LFV *ratios* via \mathbf{M}_ν !

CDF's W-mass first hint for this triplet with $O(100 \text{ GeV})$ mass? [Heeck, 2204.10274]



$$\Rightarrow (M_\nu)_{\alpha\beta} \simeq y_{\alpha\beta} \frac{2\mu v^2}{M_\Delta^2} \quad \& \quad \text{BR}(\ell_\alpha \rightarrow \ell_i \ell_j \bar{\ell}_k) \propto |(M_\nu)_{\alpha k}|^2 |(M_\nu)_{ij}|^2.$$

Normal hierarchy, $\alpha_1, \alpha_2 : (M_\nu)_{e\mu} \sim 0$.

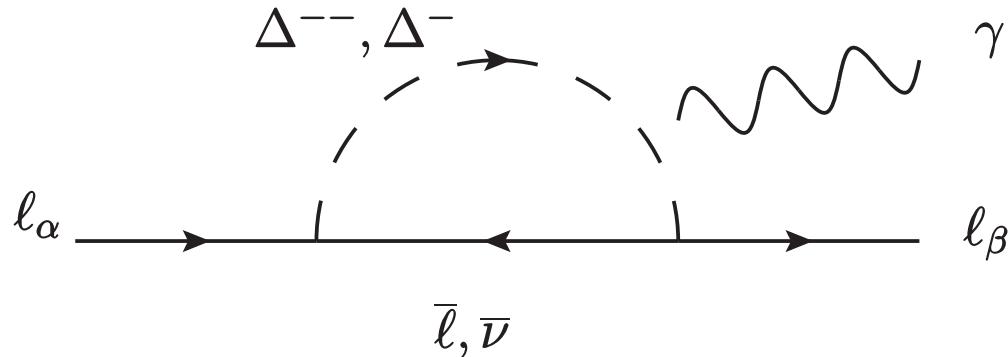


$$\Rightarrow (M_\nu)_{\alpha\beta} \simeq y_{\alpha\beta} \frac{2\mu v^2}{M_\Delta^2} \quad \& \quad \text{BR}(\ell_\alpha \rightarrow \ell_i \ell_j \bar{\ell}_k) \propto |(M_\nu)_{\alpha k}|^2 |(M_\nu)_{ij}|^2.$$

Scalar-triplet seesaw

$$(M_\nu)_{\alpha\beta} \simeq y_{\alpha\beta} \frac{2\mu v^2}{M_\Delta^2} \quad \& \quad BR(\ell_\alpha \rightarrow \ell_i \ell_j \bar{\ell}_k) \propto |y_{\alpha k}|^2 |y_{ij}|^2 / M_\Delta^4.$$

- But at loop level:



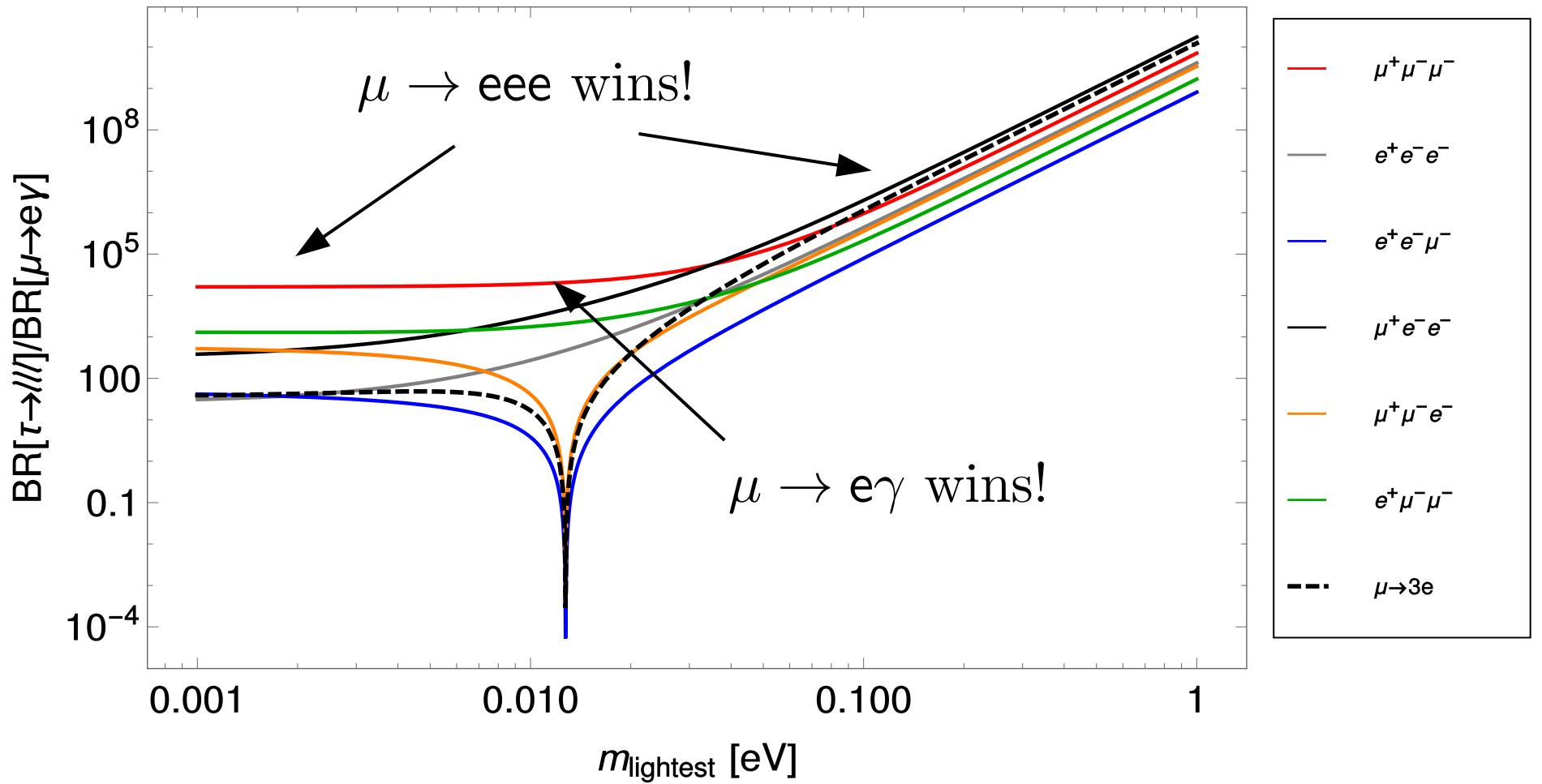
$$BR(\ell_\alpha \rightarrow \ell_\beta \gamma) \propto \frac{|(y^\dagger y)_{\alpha\beta}|^2}{M_\Delta^4}.$$

[Pich, Santamaria, Bernabeu, '84]

- $\mu \rightarrow 3e$ could be 0, but $\mu \rightarrow e\gamma$ cannot (since θ_{13}).
[Chakrabortty++, 1204.1000]
- Prediction:

$$BR(\tau \rightarrow \mu\gamma) \simeq 23 \quad BR(\tau \rightarrow e\gamma) \simeq 3.5 \quad BR(\mu \rightarrow e\gamma).$$

Normal hierarchy, $\alpha_1, \alpha_2 : (M_\nu)_{e\mu} \sim 0$.



Prediction of LFV ratios via M_ν !

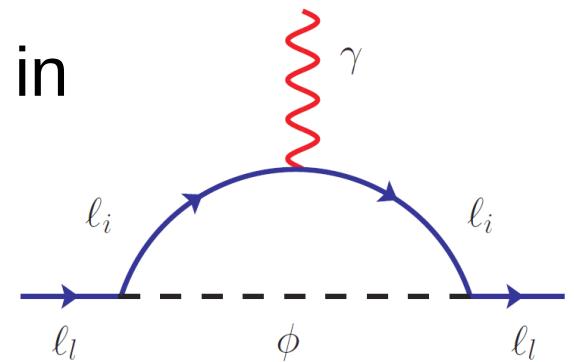
Seesaws and LFV

- Seesaws make different predictions for LFV *ratios*.

Model	$\mu \rightarrow eee$	$\mu N \rightarrow eN$	$\frac{\text{BR}(\mu \rightarrow eee)}{\text{BR}(\mu \rightarrow e\gamma)}$	$\frac{\text{CR}(\mu N \rightarrow eN)}{\text{BR}(\mu \rightarrow e\gamma)}$
Type-I seesaw	Loop*	Loop*	$3 \times 10^{-3} - 0.3$	0.1–10
Type-II seesaw	Tree	Loop	$(0.1 - 3) \times 10^3$	$\mathcal{O}(10^{-2})$
Type-III seesaw	Tree	Tree	$\approx 10^3$	$\mathcal{O}(10^3)$

[Calibbi & Signorelli, 1709.00294]

- (Muon) LFV experiments could distinguish seesaws.
- Could even access neutrino parameters (phases etc.).
- **Cannot** explain $(g-2)_\mu$ (or give large muon EDM).
- $(g-2)_\mu$ surprisingly difficult, wrong sign also in scotogenic model & Zee-Babu model, and often killed by $\mu \rightarrow e\gamma$.



Zee model

- 2HDM plus charged singlet scalar η^+ .

$$\mathcal{L} \supset f_{ij} L_i \epsilon L_j \eta^+ + \mu H_1 \epsilon H_2 \eta^- + \text{h.c.}$$

- Radiative neutrino mass:

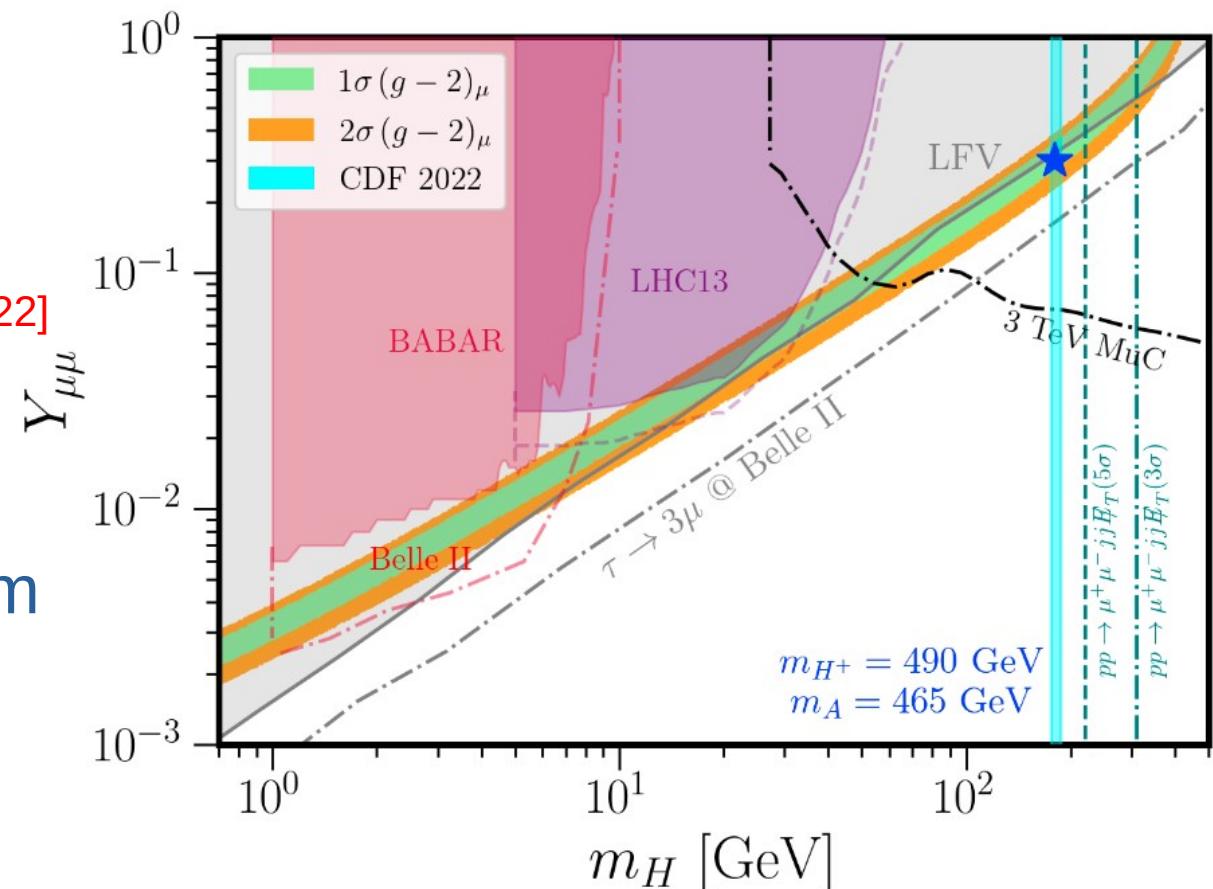
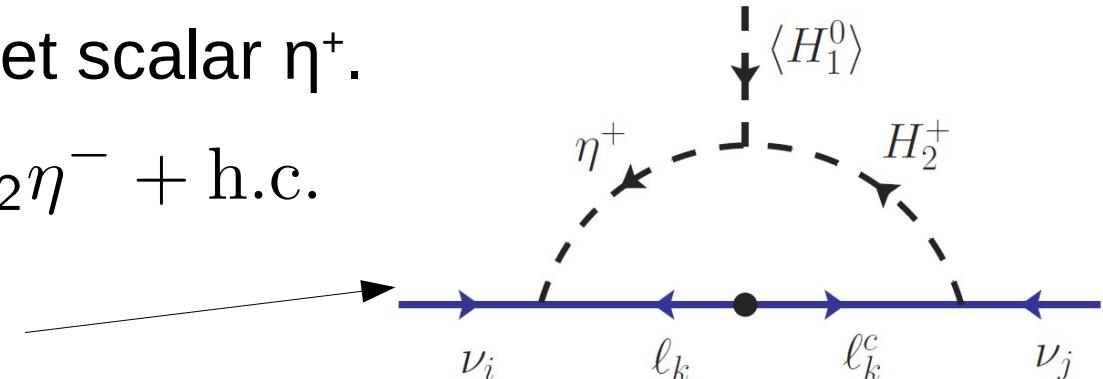
- Can explain $(g-2)_\mu$.

- Can also explain CDF's M_W !

[Chowdhury, Heeck, Thapa, Saad, '22]

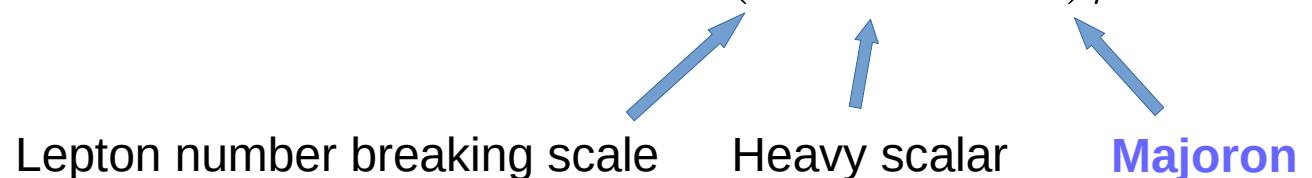
- Zee model can also yield testable muonium-antimuonium conversion!

[Heeck & Thapa, in progress]



Majoronic seesaw

- SM + 3 singlets N_R + new scalar $\sigma = (f + \sigma^0 + iJ)/\sqrt{2}$.



[Chikashige, Mohapatra, Peccei, '81; Schechter, Valle, '82]

- Break $U(1)_L$ spontaneously:

$$L = - \underbrace{\bar{L}_y H N_R}_{\text{Lepton number breaking scale}} - \frac{1}{2} \overbrace{N_R^c}^{\text{Heavy scalar}} \kappa \sigma N_R + \text{h.c.}$$

$$m_D = \frac{y}{\sqrt{2}} v \quad M_R = \frac{\kappa f}{\sqrt{2}}$$

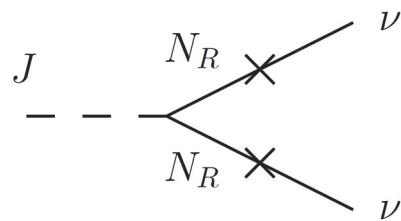
$$\longrightarrow \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

- For $M_R \gg m_D$: $M_\nu \simeq -m_D M_R^{-1} m_D^T$

$$\simeq 1 \text{ eV} \left(\frac{m_D}{100 \text{ GeV}} \right)^2 \left(\frac{10^{13} \text{ GeV}}{M_R} \right).$$

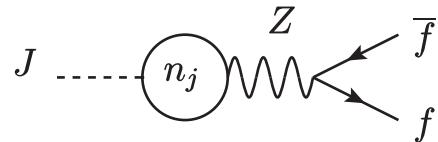
Majoron couplings

- Tree-level coupling only to neutrinos:

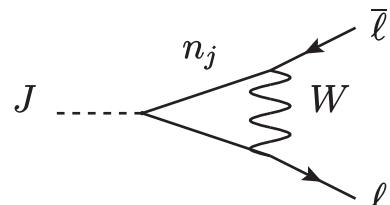


$$\frac{ij}{2f} \bar{\nu}_\alpha^c \gamma_5 (m_D M_R^{-1} m_D^\top)_{\alpha\beta}^* \nu_\beta = - \underbrace{\frac{ij}{2f} \sum_k \bar{\nu}_k \gamma_5 m_k \nu_k}_{\text{Too small for lab}}$$

- One loop:



$$\frac{ij}{f} \bar{f} \gamma_5 f \frac{m_f T_3^f}{8\pi^2 v^2} \text{tr} (m_D m_D^\dagger)$$



$$\frac{ij}{f} \bar{\ell}_\alpha \left(\frac{m_\beta}{8\pi^2 v^2} P_R - \frac{m_\alpha}{8\pi^2 v^2} P_L \right) \ell_\beta \left(m_D m_D^\dagger \right)_{\alpha\beta}$$

[JH, Garcia-Cely, JHEP '17; see also Pilaftsis '94]

Too small for lab

Off-diagonal!

Properties

- Crucial observation: the two matrices are independent!

$$\{m_D, M_R\} \leftrightarrow \{M_\nu = -m_D M_R^{-1} m_D^T, m_D m_D^\dagger\}.$$

[Davidson, Ibarra, JHEP '01]

- $J\bar{\ell}\ell'$ coupling can be *large* and of arbitrary structure.
- Similar couplings arise for familons or flavor Z' .

[Wilczek, '82; Reiss, '82; Grinstein, Preskill, Wise, 85; ...]

- Experimental signature depends on J decay channel:

$$\ell \rightarrow \ell' J, \quad J \rightarrow \text{inv}, \ell'' \ell''', \gamma\gamma, \dots$$



$[\mu \rightarrow e J, J \rightarrow \gamma\gamma]$
MEG, 2005.00339]

[JH, Rodejohann, PLB '18;
Bauer et al., PRL '20;
Cornella et al., JHEP '20;
Cheung++, JHEP '21]

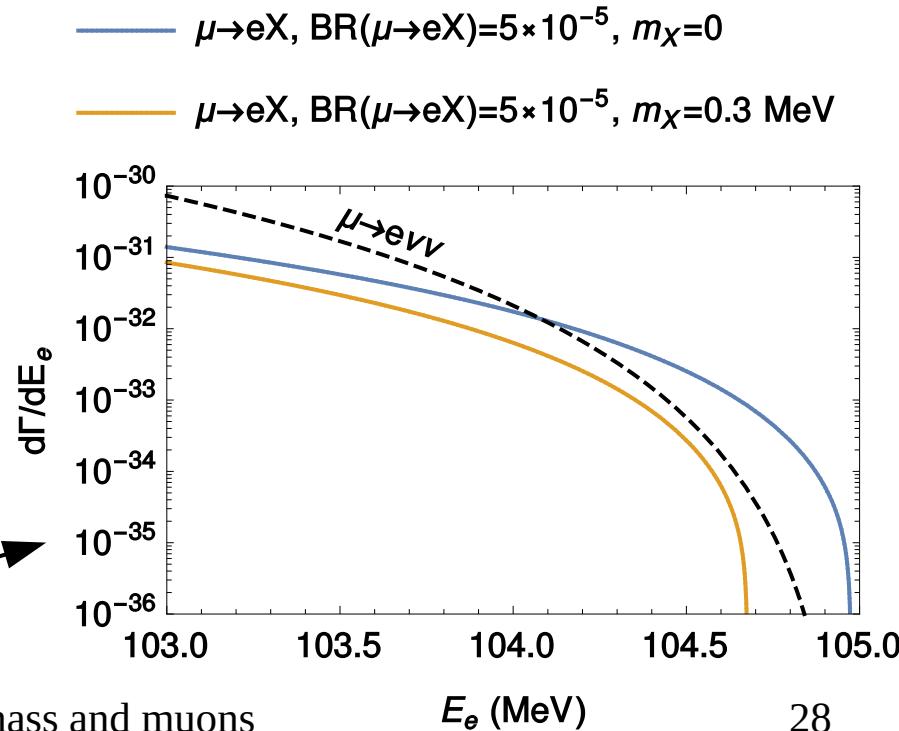
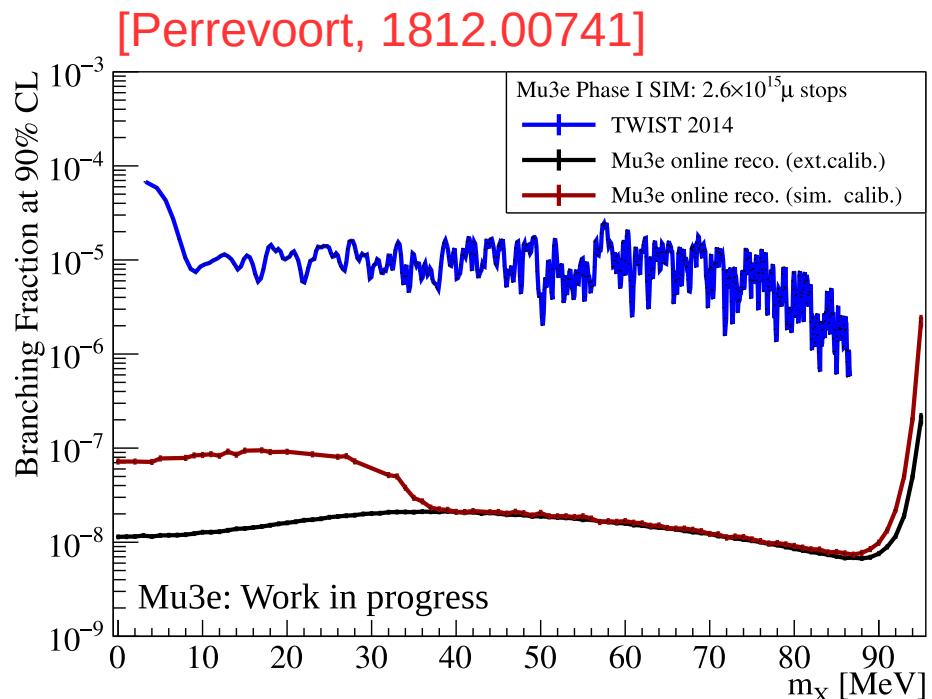
$\mu \rightarrow e J$

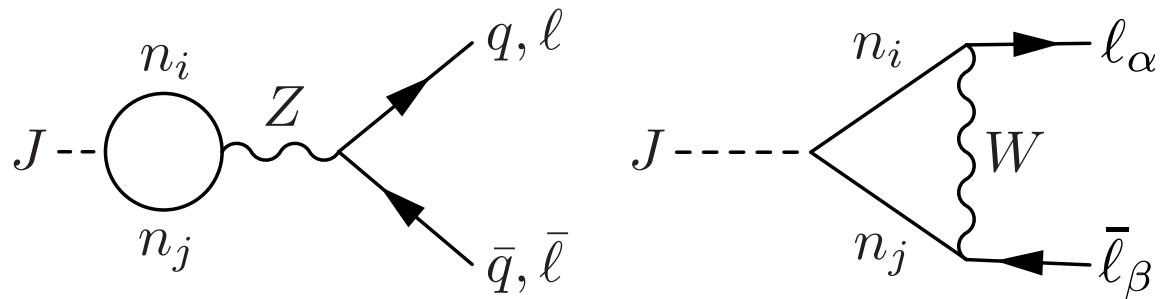
See talk by
Perrevoort.

- Electron *line* on top of Michel spectrum.
- Good prospects @ Mu3e.
- In progress: signal in $\mu \rightarrow e$ conversion exps. COMET, Mu2e(-II).
 - Many muons!
 - Nuclear recoil: E_e up to m_μ .
 - Suppression of tail...

[Garcia i Tormo++, PRD '11;
Uesaka, 2005.07894]

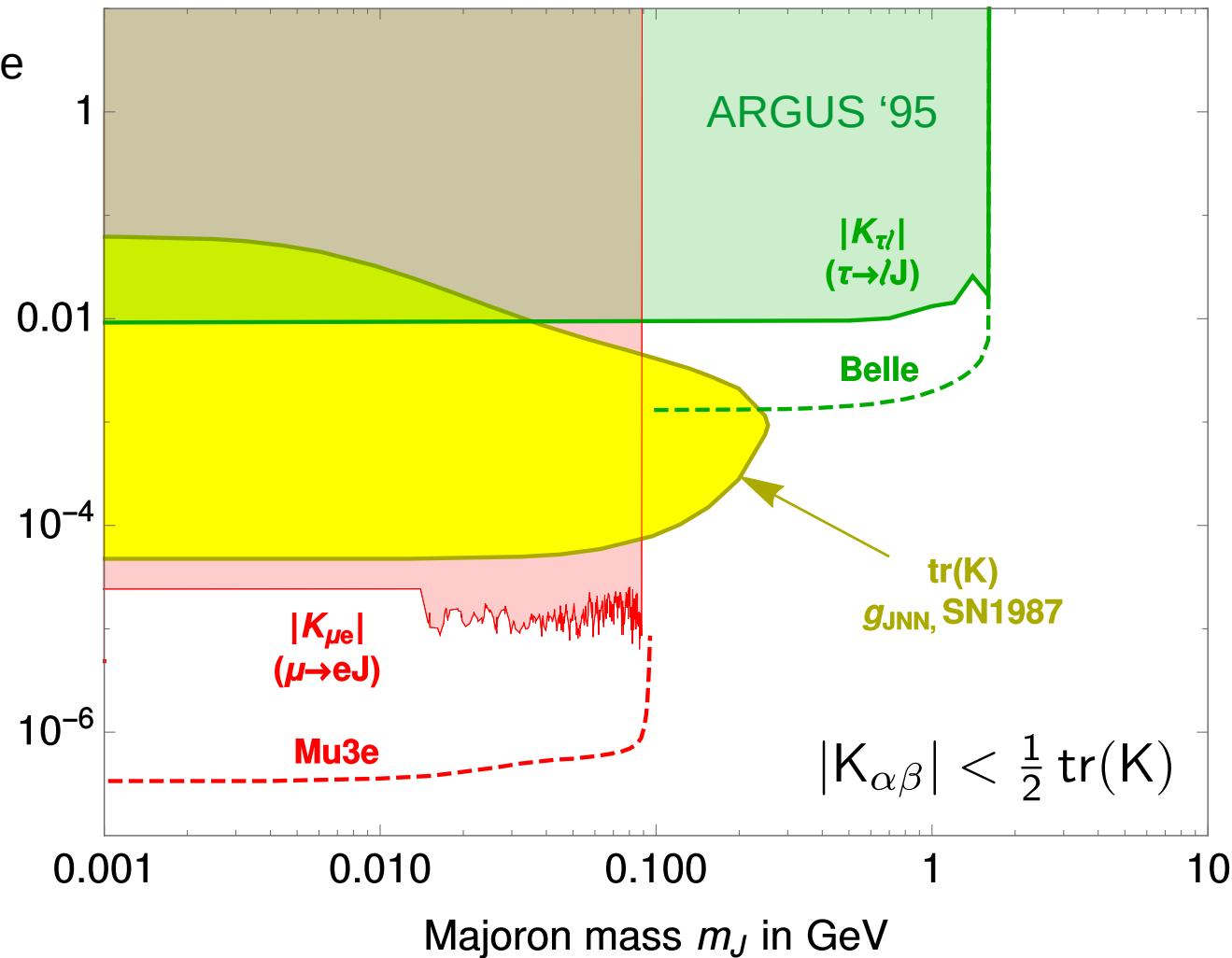
[JH++, Mu2e-II Snowmass LOI]





Limit on effective coupling

$$K \equiv \frac{m_D m_D^\dagger}{f v}$$



[JH, Garcia-Cely, JHEP '17]

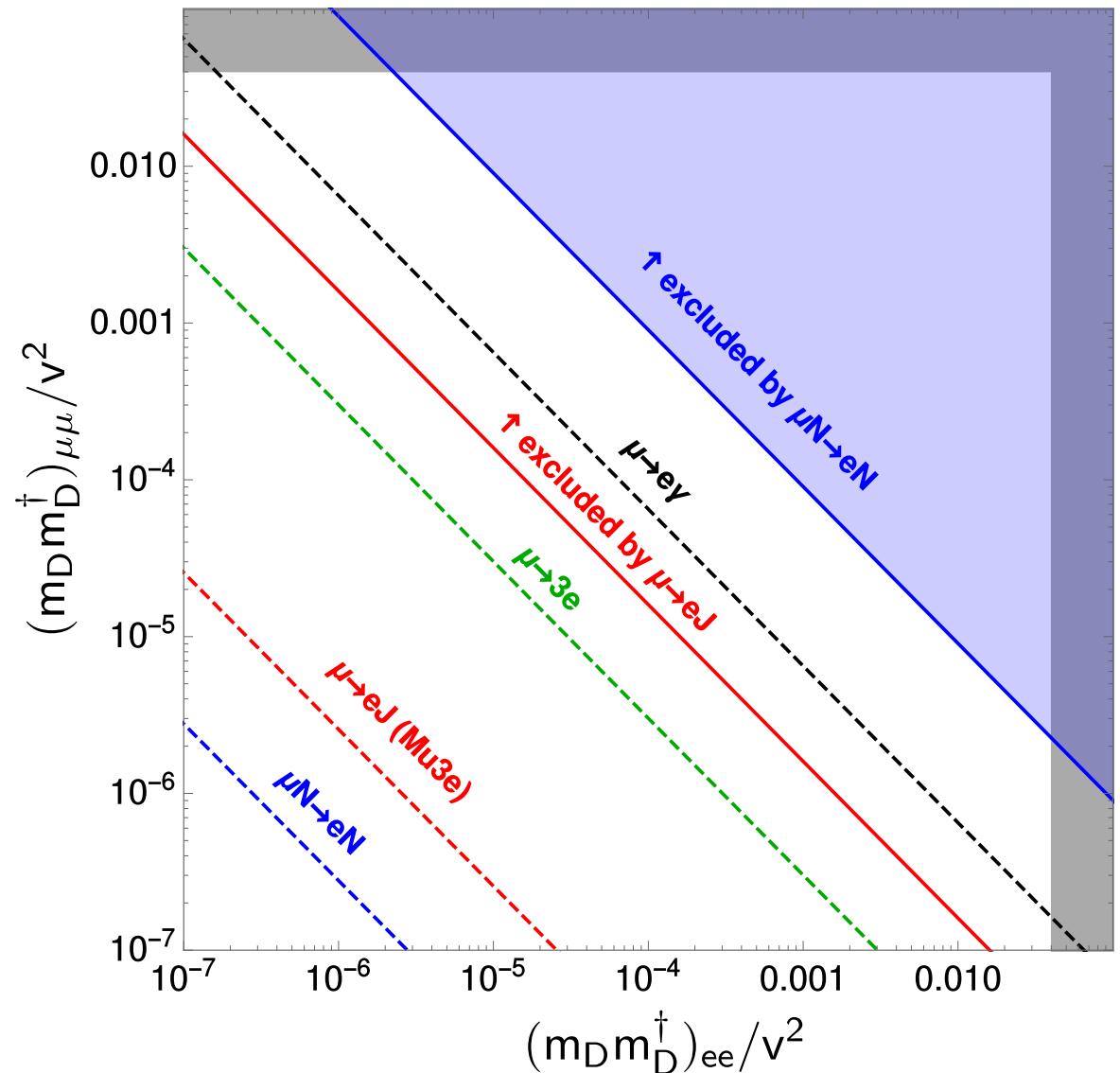
$$M_R = f = 1 \text{ TeV}$$

$$(m_D m_D^\dagger)_{e\mu} = [(m_D m_D^\dagger)_{ee} (m_D m_D^\dagger)_{\mu\mu}]^{1/2}$$

- Comparison of Majoron and non-Majoron limits.

[from Coy & Frigerio, PRD '19]

- $m_D M_R^{-2} m_D^\dagger$ vs. $\frac{m_D m_D^\dagger}{f}$.
- Sterile neutrinos modify EWPD & LFV.
- $\frac{\Gamma(\ell \rightarrow \ell' \gamma)}{\Gamma(\ell \rightarrow \ell' J)} \simeq 2\pi\alpha \frac{m_\ell^2}{M_R^2} \frac{f^2}{M_R^2}$.
- Majoron wins for $f \sim M_R$.
- $\ell \rightarrow \ell' + J$ possible!
- Together with LFV in μ ?



[JH, Patel, PRD '19]

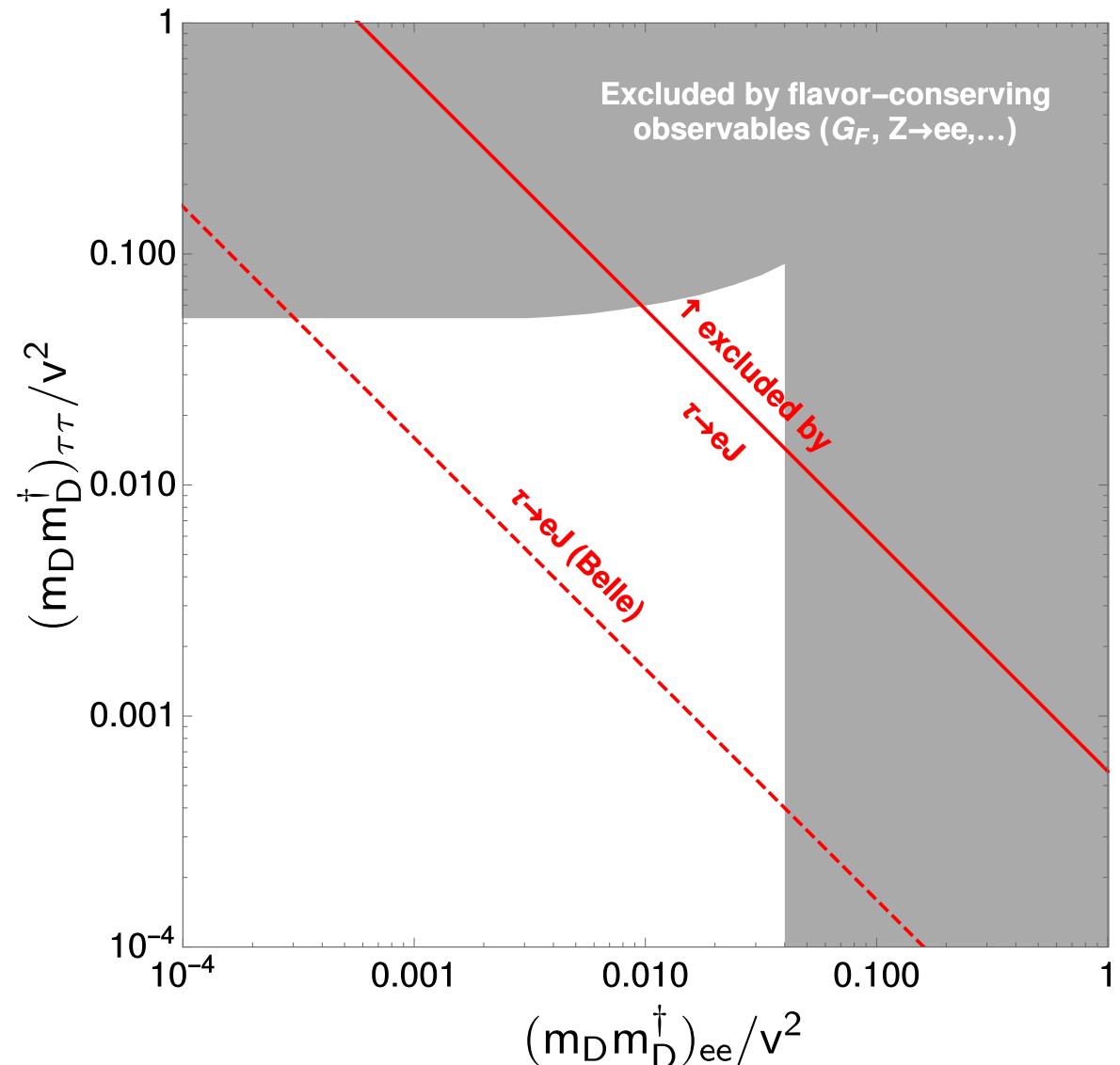
$$M_R = f = 1 \text{ TeV}$$

$$(m_D m_D^\dagger)_{e\tau} = [(m_D m_D^\dagger)_{ee} (m_D m_D^\dagger)_{\tau\tau}]^{1/2}$$

- Comparison of Majoron and non-Majoron limits.

[from Coy & Frigerio, PRD '19]

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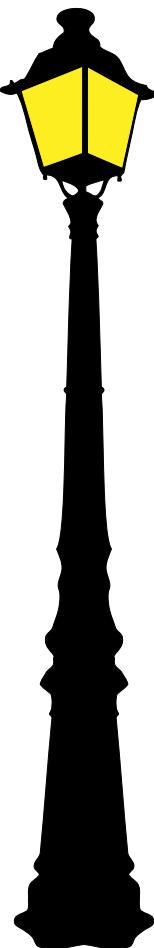


[JH, Patel, PRD '19]

Summary

- Neutrino mass models *qualitatively* predict CLFV, sometimes quantitative predictions of *ratios*.
- Expect first observations in muon experiments, channel ($\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu N \rightarrow eN$, $\bar{\mu}e \rightarrow \bar{\mu}e$, $\mu \rightarrow eJ$) depends on model.
- Observations complement neutrino experiments.
- Some models can explain $(g-2)_\mu$ anomaly.
- Light new physics open new avenues.

Explore every corner of our lamppost!



Backup

Effective field theory view

- SM symmetry: $G = U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$.
- Effective field theory with Majorana ν :

$$L = L_{\text{SM}} + \frac{M_\nu}{\Lambda} \underbrace{\overline{LLHH}}_{\substack{\text{conserves } G \\ \text{violates } G}} + \sum_j \frac{\mathcal{O}_j}{\Lambda^2} + \sum_j \frac{\mathcal{O}'_j}{\Lambda^3} + \sum_j \frac{\mathcal{O}''_j}{\Lambda^4} + \dots$$

could conserve G or subgroup
 \Rightarrow ‘weird’ channels dominate!?

Upcoming CLFV

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LORENZO CALIBBI and GIOVANNI SIGNORELLI

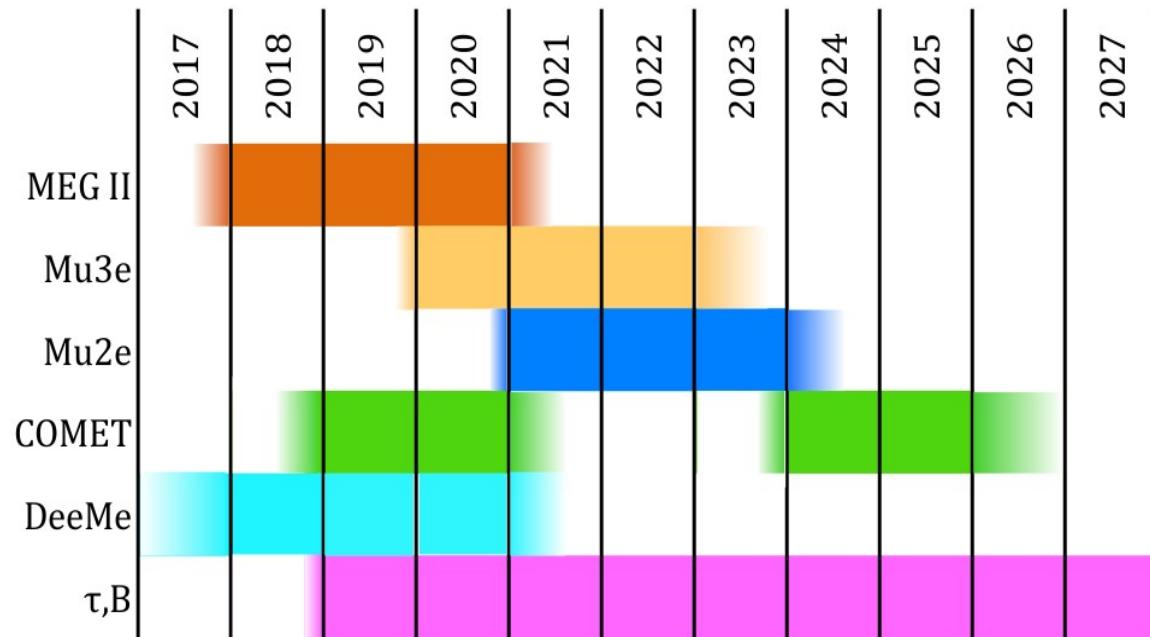
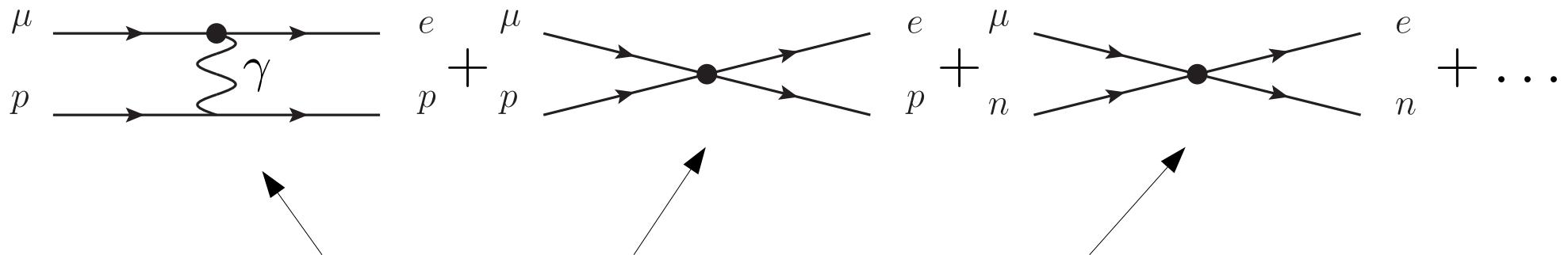


Figure 47. – Projected time lines for different projects searching for CLFV decays. MEG II is expected to start data taking in 2018 after an engineering run in 2017; Mu3e magnet and detectors are expected at the end of 2019; Mu2e foresees three years of data taking starting in 2021; COMET Phase-I is expected to start commissioning and data taking in 2018 for two-three years, followed by a stop to develop and deploy the beamline and detectors for Phase-II; DeeMe is expected to start soon and take data with graphite and silicon carbide targets in sequence; Belle II is schedule to start data taking at end 2018.

[Calibbi & Signorelli, 1709.00294]

The inverse problem

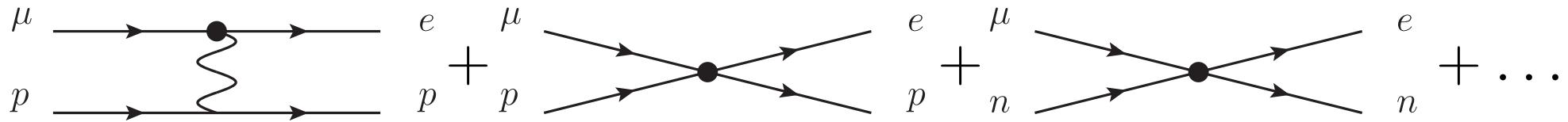
- If we see CLFV, can we pin down the underlying operator?
 - In many cases: Yes! (e.g. $\mu \rightarrow e y \leftrightarrow$ dipole)
 - $\mu \rightarrow e$ conversion in nucleus: No!



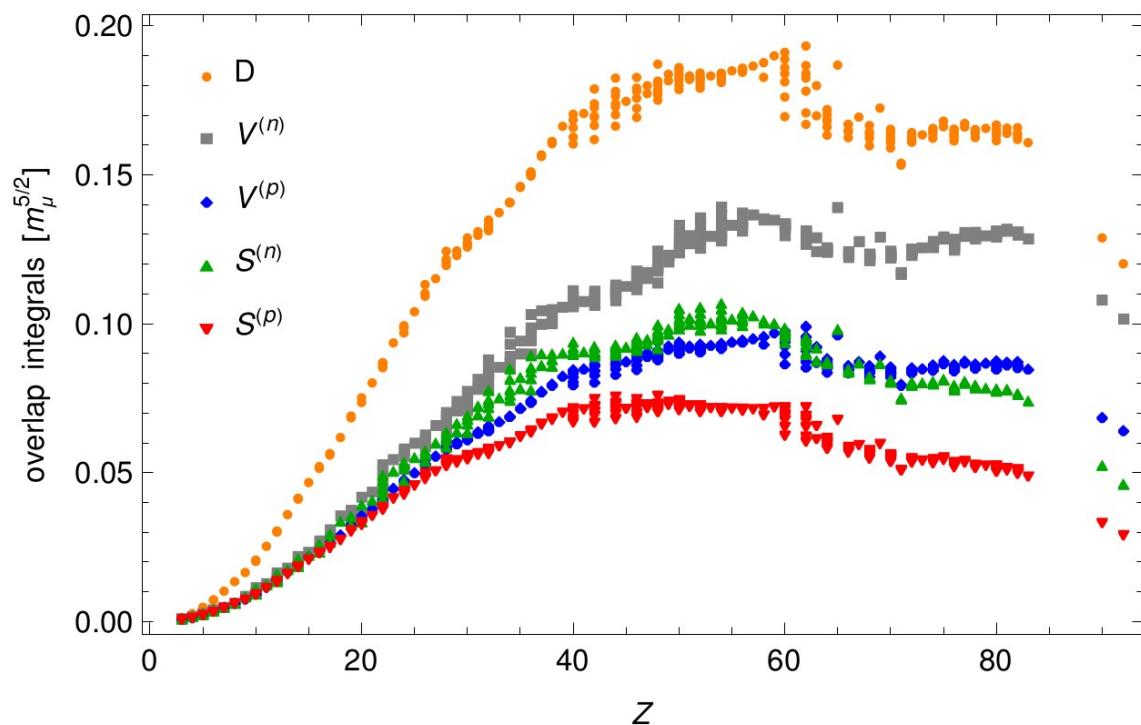
Relative contributions depend on nucleus: Z, N, spin!

- Need to observe $\mu \rightarrow e$ conversion in different nuclei!
[Kitano, Koike, Okada, PRD '07; Cirigliano++, PRD '09; Davidson++, '18]

$\mu \rightarrow e$ conversion



- Assuming spin-*independent* conversion:



$$\text{BR}_{\text{SI}} = \frac{32G_F^2}{\Gamma_{\text{capture}}} [|v \cdot C_L|^2 + |v \cdot C_R|^2]$$

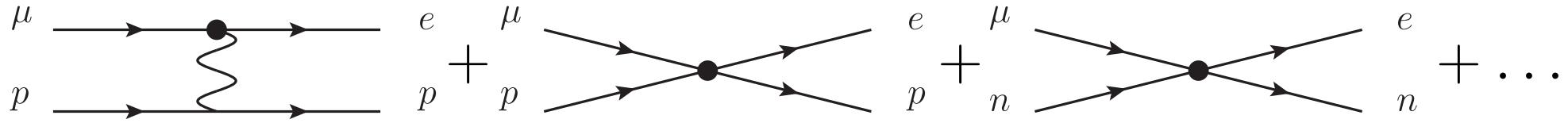
Overlap integrals

Wilson coefficients

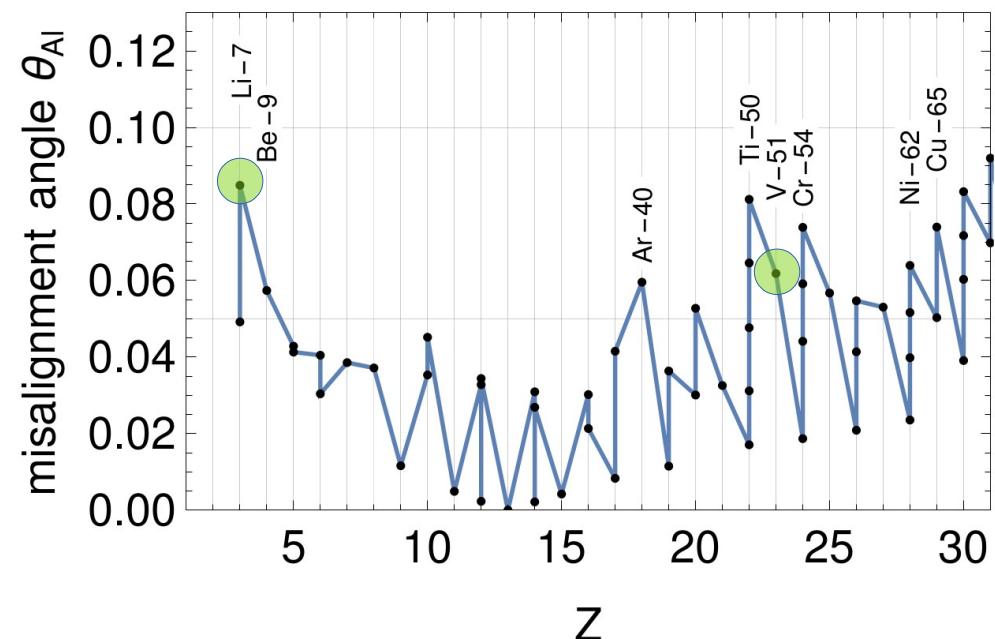
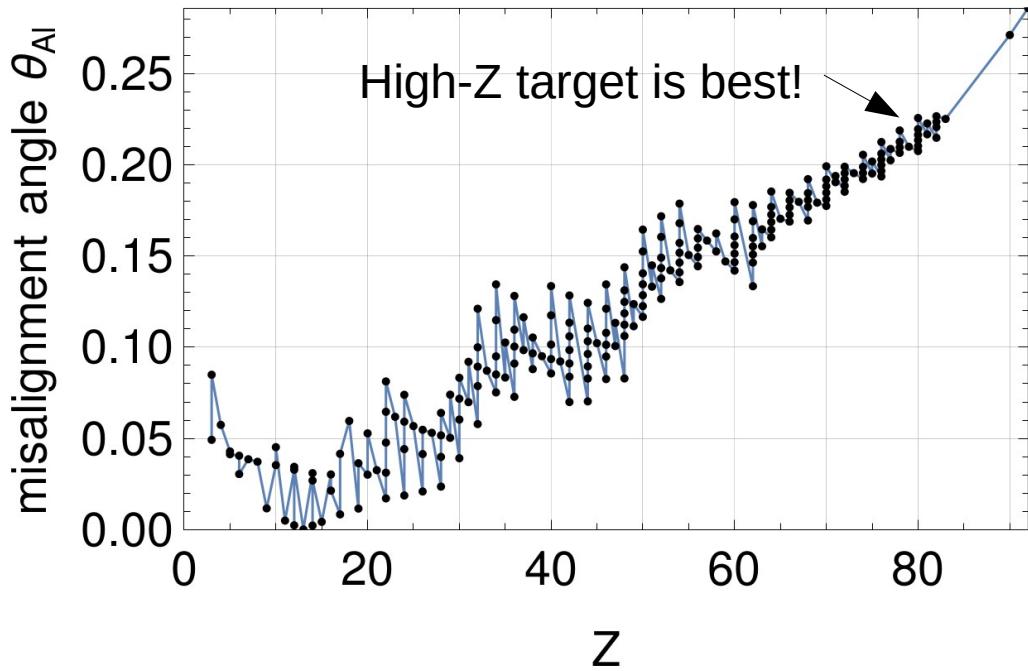
$$v \equiv \left(\frac{D}{4}, V^{(p)}, S^{(p)}, V^{(n)}, S^{(n)} \right)$$

To measure the Wilson coefficients, use nuclei whose v are maximally misaligned.
 [Davidson, Kuno, Yamanaka, PLB '19]

$\mu \rightarrow e$ conversion



- Misalignment with aluminium (target in COMET & Mu2e):



- At low Z, Li-7 and V-51 can distinguish proton/neutron.

[Davidson, Kuno, Yamanaka, PLB '19; Heeck, Szafron, Uesaka, NPB '22]

Pseudo-Goldstone

- Spontaneous global $U(1)$ breaking gives $m_J = 0$.
- Non-zero mass from:
 - Breaking by gravity, e.g. wormholes,
$$m_J \sim M_{Pl} \exp[-\mathcal{O}(M_{Pl}/f)] .$$
 - Anomalies, e.g. if $U(1)_{B-L} = U(1)_{PQ}$.
[Mohapatra, Senjanovic '83; Langacker, Peccei, Yanagida '86; SMASH '16]
 - Explicit breaking, e.g. $\Delta V = \frac{1}{2} m_J^2 J^2$.